UNIVERSIDADE FEDERAL DO ESPÍRITO SANTO

DEPARTAMENTO DE ENGENHARIA ELÉTRICA (DEE)

[MOD-MTL] Analysis of Multiconductor Transmission Lines

Clayton Paul's Time & Frequency Domain Modelling, Triple Microstrip (Fig. 5.32) and Directional Beat

Prof. Marcelo Eduardo Vieira Segatto

LabTel — Laboratório de Telecomunicações | IN-Foton





Module Outline [MOD-MTL] — Bonus Material

1. Theoretical Foundations of Multiconductor Lines (MTL)
2. Dimensioning of the Triple Microstrip (Figure 5.32)
3. 2D Electrostatic Solution and Normal Modes
4. Time-Domain Pulse Propagation and Directional Beat
5. 60 Hz MTL Study: Overhead Distribution Lines (13.8 kV)
6. Conclusions and Open Science

Time-Domain Telegrapher's Equations

For a uniform waveguiding system with n active conductors parallel to the z-axis and a ground reference (0):

Matrix Telegrapher's Equations (Clayton Paul)

$$\frac{\partial \mathbf{v}(z,t)}{\partial z} = -\mathbf{R}\,\mathbf{i}(z,t) - \mathbf{L}\,\frac{\partial \mathbf{i}(z,t)}{\partial t}$$

$$\frac{\partial \mathbf{i}(z,t)}{\partial z} = -\mathbf{G}\,\mathbf{v}(z,t) - \mathbf{C}\,\frac{\partial \mathbf{v}(z,t)}{\partial t}$$

- $\mathbf{v}(z,t) = [v_1, \ldots, v_n]^T$: nodal line voltages referenced to common ground;
- $\mathbf{i}(z,t) = [i_1, \ldots, i_n]^T$: longitudinal conduction currents;
- $[\mathbf{R}], [\mathbf{L}], [\mathbf{G}], [\mathbf{C}]$: symmetric $n \times n$ per-unit-length (p.u.l.) parameter matrices.

Phasor Formulation and Matrix Wave Equations

In the angular frequency domain $\omega = 2\pi f$ ($\mathbf{V}(z)e^{j\omega t}$, $\mathbf{I}(z)e^{j\omega t}$):

$$\frac{d\mathbf{V}(z)}{dz} = -\mathbf{Z}\,\mathbf{I}(z), \quad \frac{d\mathbf{I}(z)}{dz} = -\mathbf{Y}\,\mathbf{V}(z)$$

with $\mathbf{Z} = \mathbf{R} + j\omega\mathbf{L}$ and $\mathbf{Y} = \mathbf{G} + j\omega\mathbf{C}$.

Second-Order Decoupled Wave Equations

$$\frac{d^2\mathbf{V}(z)}{dz^2} = \mathbf{ZY}\,\mathbf{V}(z), \quad \frac{d^2\mathbf{I}(z)}{dz^2} = \mathbf{YZ}\,\mathbf{I}(z)$$

- In inhomogeneous media (microstrip), $\mathbf{LC} \neq \mu\varepsilon\mathbf{I}$;
- The matrix products $\mathbf{ZY}$ and $\mathbf{YZ}$ are not identity scalars;
- Guided waves propagate as quasi-TEM modes with **distinct modal phase velocities**.

Modal Decoupling and Eigenvector Transformations

Linear transformations to uncoupled modal coordinates:

$$\mathbf{V}(z) = \mathbf{T}_V\, \mathbf{V}_m(z), \quad \mathbf{I}(z) = \mathbf{T}_I\, \mathbf{I}_m(z)$$

where $\mathbf{T}_V$ diagonalizes $\mathbf{ZY}$ and $\mathbf{T}_I$ diagonalizes $\mathbf{YZ}$ ($\mathbf{T}_I^T = \mathbf{T}_V^{-1}$):

Eigenvalues and Modal Phase Velocities

$$\mathbf{T}_V^{-1}(\mathbf{ZY})\mathbf{T}_V = \mathrm{diag}(\gamma_1^2, \gamma_2^2, \ldots, \gamma_n^2)$$

$$v_m = \frac{1}{\sqrt{\lambda_m(\mathbf{LC})}}, \quad \beta_m = \omega\sqrt{\lambda_m(\mathbf{LC})}$$

Modal phase dispersion induces pulse distortion and Far-End Crosstalk (FEXT).

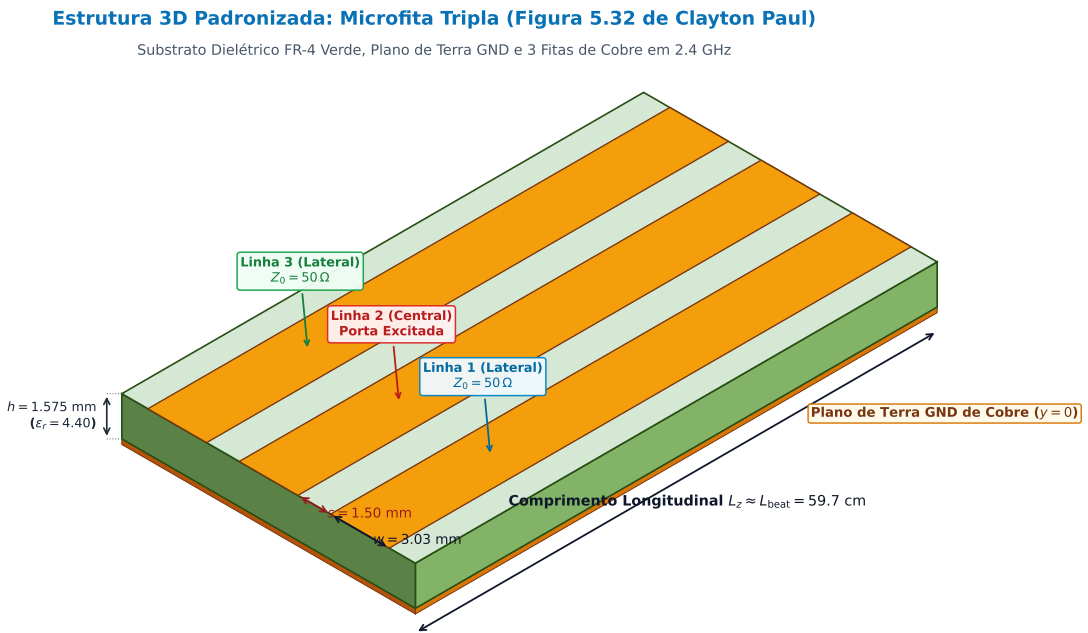
Canonical Geometry of Figure 5.32 at 2.4 GHz

Three symmetric coplanar strips over an FR-4 substrate with bottom ground plane:

Physical Parameter	**Symbol**	**Design Value**
Operating Frequency	f_0	2.40 GHz ($\lambda_0 = 125.0$ mm)
Relative Permittivity	ε_r	4.40 (Standard RF FR-4)
Substrate Thickness	h	1.575 mm (62.0 mils)
Strip Width	w	3.03 mm ($w/h = 1.924 \implies Z_{0,\text{iso}} = 50.0\,\Omega$)
Edge-to-Edge Spacing	s	1.50 mm ($s/h = 0.952$, strong coupling)
Spatial Beat Length	L_{beat}	596.6 mm (≈ 59.7 cm)

- Each isolated strip is matched to $Z_0 = 50\,\Omega$ per specification;
- Spacing $s = 1.5$ mm establishes balanced bilateral mutual coupling.

Standardized 3D Topology (LabTel / IN-Foton)



Triple microstrip: FR-4 substrate ($\varepsilon_r = 4.40, h = 1.575$ mm), copper strips ($w = 3.03$ mm, $s = 1.50$ mm), and continuous ground plane.

Numerical Extraction via Clayton Paul's Moment Method

Integration of the infinite series of dielectric image charges (Clayton Paul, Sec. 5.3.1.2):

$$\phi(d) = \frac{(1+k)q}{4\pi\varepsilon_0\varepsilon_{r,\text{eff},\infty}} \sum_{n=1}^{\infty} (-1)^{n+1} k^{n-1} \ln\left(\frac{d^2 + (2nh)^2}{d^2}\right)$$

with $k = (\varepsilon_r - 1)/(\varepsilon_r + 1)$ and closed kernel $F(u, a)$.

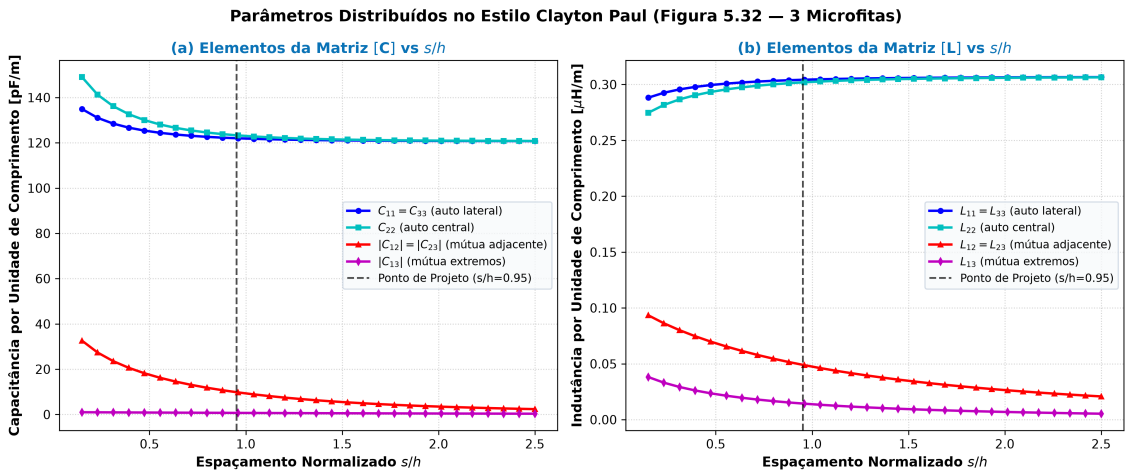
Extracted Numerical Matrices (UFES / LabTel)

$$[\mathbf{C}] = \begin{bmatrix} 122.46 & -9.97 & -0.70 \\ -9.97 & 123.80 & -9.97 \\ -0.70 & -9.97 & 122.46 \end{bmatrix} \ [\text{pF/m}]$$

$$[\mathbf{L}] = \begin{bmatrix} 0.3033 & 0.0492 & 0.0145 \\ 0.0492 & 0.3009 & 0.0492 \\ 0.0145 & 0.0492 & 0.3033 \end{bmatrix} \ [\mu\text{H/m}]$$

Notice $|C_{12}| \gg |C_{13}|$ and $L_{12} \gg L_{13}$, confirming dominant adjacent coupling.

Coupling Behaviour as a Function of Spacing s/h



Replication of Figs. 5.34 and 5.35 from Clayton Paul: exponential decay of mutual capacitance and inductance as strip spacing increases.

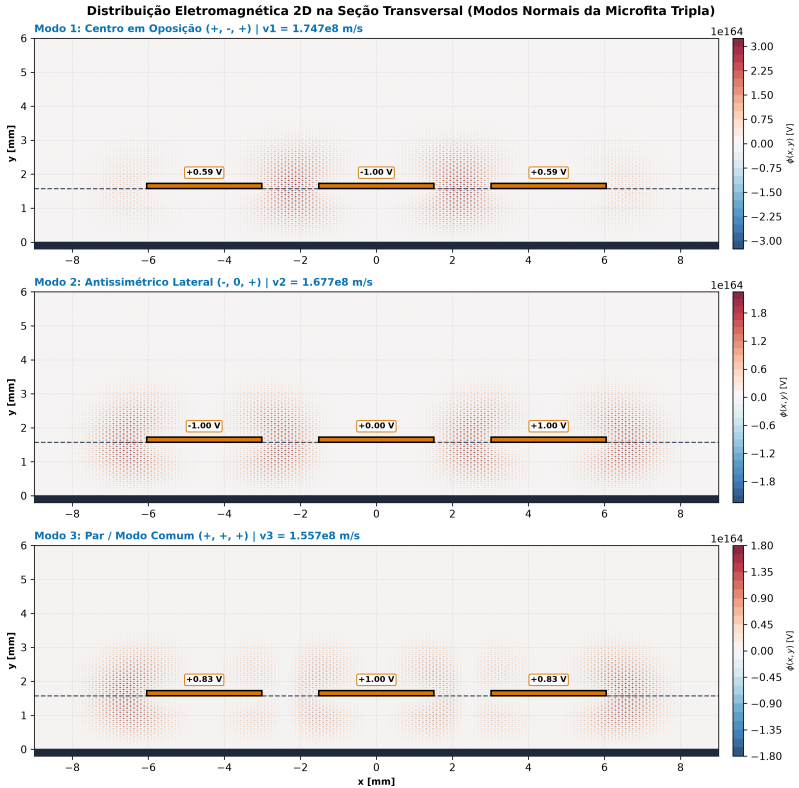
2D Cross-Sectional Finite-Difference Mesh

Solving the 2D Poisson/Laplace equation over the transverse grid (x, y):

$$\nabla \cdot [\varepsilon_r(x, y) \nabla \Phi(x, y)] = 0$$

- Boundary Conditions: $\Phi(x, 0) = 0$ (ground plane), $\Phi = 0$ at outer open boundaries (Dirichlet/Neumann);
- Three Fundamental Orthogonal Modes:
  1. **Mode 1 (Odd-Symmetric):** $\mathbf{M}_1 = [1, 0, -1]^T$
  2. **Mode 2 (Even-Symmetric):** $\mathbf{M}_2 = [1, -\sqrt{2}, 1]^T$
  3. **Mode 3 (In-Phase Common):** $\mathbf{M}_3 = [1, \sqrt{2}, 1]^T$

Transverse Equipotentials and Electric Fields



Spatial structure of the 3 fundamental modes: contour lines $\Phi(x, y)$ and vector arrows $\vec{E}(x, y)$.

Modal Parameters at 2.4 GHz

Analytical and numerical eigenvalues from $[\mathbf{L}][\mathbf{C}]$:

Mode	**Effective ε_{eff}**	**Phase Velocity v_m**	**Wavenumber β_m**	**Wavelength λ_g**	**Symmetry Type**
Modo 1	2.94	1.750×10^8 m/s	86.29 rad/m	72.8 mm	Odd / Antisymmetric
Modo 2	3.20	1.676×10^8 m/s	90.01 rad/m	69.8 mm	Even / Centre Inverted
Modo 3	3.71	1.558×10^8 m/s	96.83 rad/m	64.9 mm	Even / All In-Phase

Dispersion Consequence

Because $\varepsilon_{\text{eff},1} \neq \varepsilon_{\text{eff},2} \neq \varepsilon_{\text{eff},3}$, a single pulse decomposes into waves with distinct velocities, generating spatial phase shift $\Delta\beta$ along z.

Coupling Dynamics and Spatial Beat

When driving Line 2 (center) with initial excitation $\mathbf{v}(0) = [0, 1, 0]^T$:

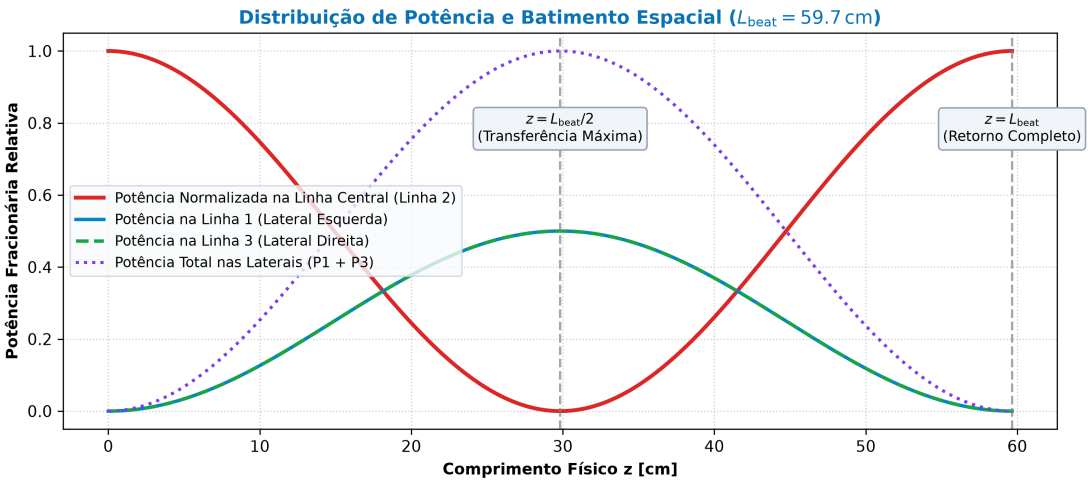
- The signal excites exclusively **Modes 1 and 3** (Mode 2 is null at centre);
- Since $\beta_1 \neq \beta_3$, progressive spatial interference develops along z.

Theoretical Spatial Beat Length (L_{beat})

$$L_{\text{beat}} = \frac{2\pi}{|\beta_3 - \beta_1|} = \frac{2\pi}{96.83 - 86.29} = 596.6 \text{ mm} \approx 59.7 \text{ cm}$$

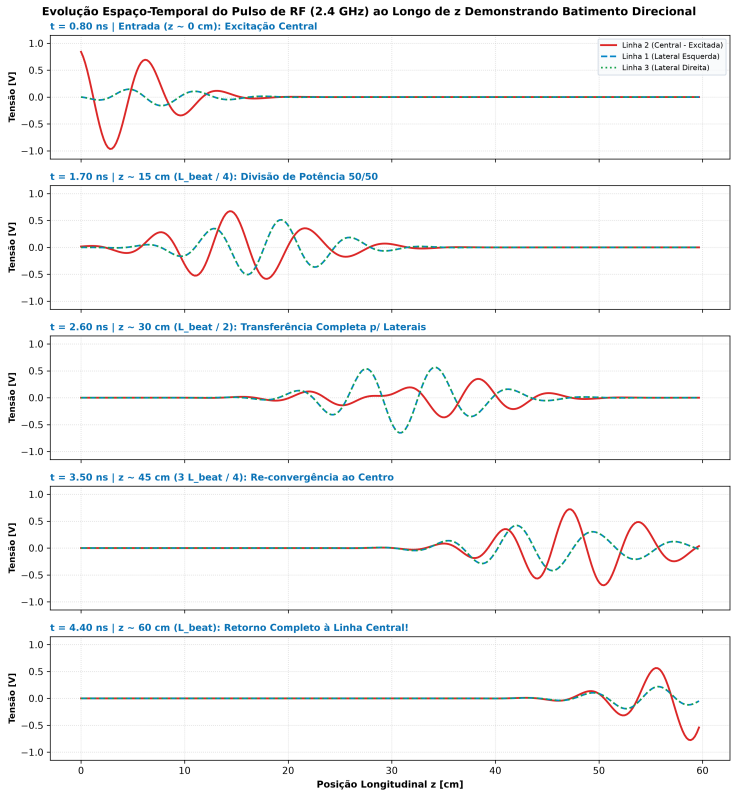
- $z = L_{\text{beat}}/2 \approx 30$ cm: Maximum energy transfer to outer lines 1 and 3 ($V_2 \approx 0$);
- $z = L_{\text{beat}} \approx 60$ cm: **Complete reconvergence of energy back to the centre line**.

Power Envelope Along the Longitudinal z-Axis



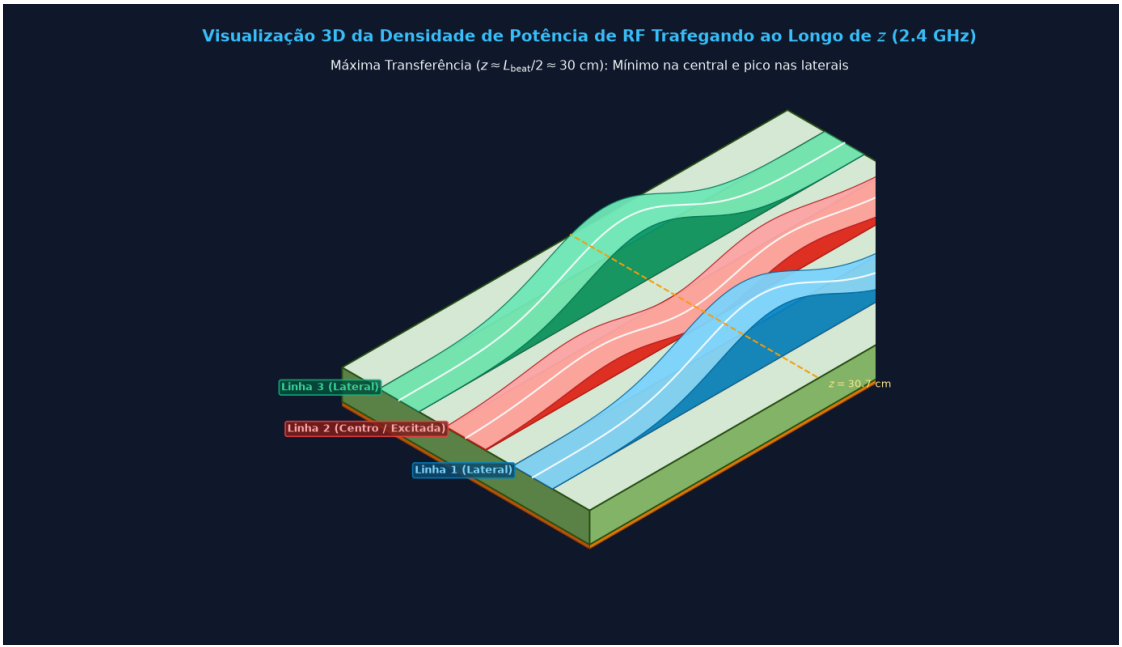
Periodic fractional power transfer illustrating the directional coupling cycle.

Time-Domain Pulse Snapshots at 2.4 GHz



Spatial voltage profiles $V_1(z), V_2(z), V_3(z)$ at 5 time steps showing pulse reconvergence.

3D Volumetric Power Density Visualization



3D volumetric RF power density ($|v_i(z,t)|^2$): the packet injected into the centre strip (red) diffracts to outer strips at $z \approx 30$ cm and fully reconverges at $z \approx 60$ cm.

Overhead Medium-Voltage Lines (13.8 kV) Under Brazilian Standards

Applying Clayton Paul's MTL formulation to 60 Hz electric power grids:

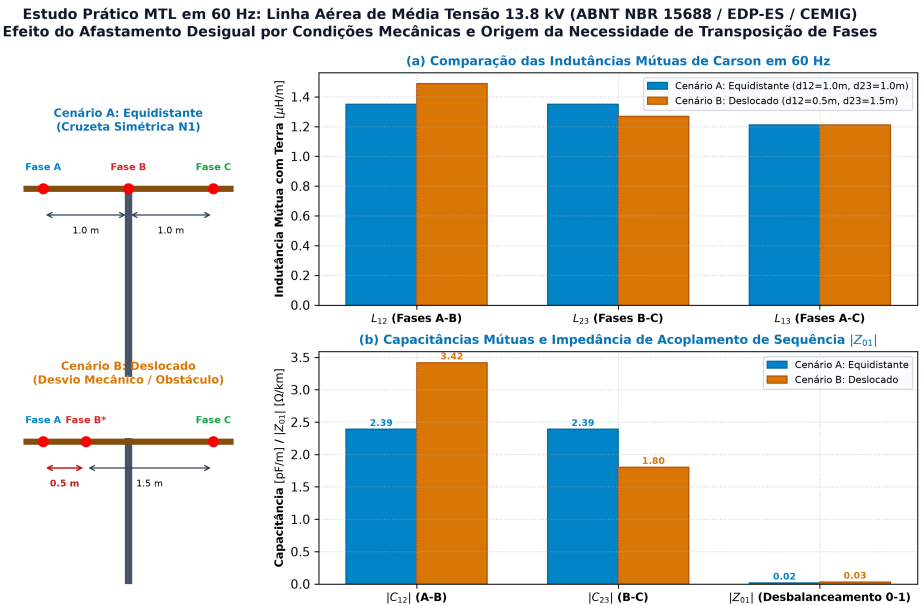
- **Standards:** ABNT NBR 15688 (bare overhead lines), NBR 14039 (13.8 kV), NBR 7270 (ACSR conductors);
- **Conductor:** 1/0 AWG ACSR "Raven"($r = 5.055$ mm, $r' = 4.47$ mm, $R_{ac} = 0.536\,\Omega/\text{km}$);
- **Average Height:** $H = 10.0$ m (11 m/12 m utility poles with 40 m span).

Carson's Earth Return Path (Clayton Paul, Sec. 4.3)

For typical soil resistivity $\rho = 100\,\Omega \cdot \text{m}$ at $f = 60$ Hz:

$$D_e = 658.87\sqrt{\frac{\rho}{f}} = 850.6 \text{ m}, \quad \Delta R_e = \pi^2 \times 10^{-7}\omega \approx 0.372\,\Omega/\text{km}$$

Mechanical Origin of Asymmetric Spacing (13.8 kV)



Urban clearance restrictions (NBR 15688), trees, or transformer mountings force asymmetrical crossarms, shifting the centre conductor closer to the left phase (Scenario B).

Numerical Matrices [L], [C] and Fortescue Symmetrical Components

Analytical comparison of distributed parameters at 60 Hz between Scenarios A and B:

Electromagnetic Parameter	**Scenario A (Equidistant)**	**Scenario B (Shifted)**	**Physical Impact**
Spacing D_{12} vs D_{23}	1.0 m / 1.0 m	0.5 m / 1.5 m	Centre cable shifted by half
Mutual Inductances L_{12}, L_{23}	1.349 / 1.349 mH/km	**1.488** / **1.268** mH/km	**Mutual asymmetry of** 14.77%
Mutual Capacitance $\lvert C_{12}\rvert$	2.390 nF/km	**3.419** nF/km	**Increase of** +43.06%
Zero Sequence Z_{00}	$1.652 + j1.899\,\Omega$/km	$1.652 + j1.913\,\Omega$/km	Earth reactance stable
Positive Sequence Z_{11}	$0.536 + j0.425\,\Omega$/km	$0.536 + j0.418\,\Omega$/km	Series reactance stable
Cross-Coupling $\lvert Z_{01}\rvert$	$\mathbf{0.017\,\Omega}$/km	$\mathbf{0.032\,\Omega}$/km	**Nearly doubles** (+88.2%)

- Physical asymmetry destroys the cyclic symmetry of $[\mathbf{Z}_{abc}]$ and $[\mathbf{Y}_{abc}]$;
- Sequence mutual coupling induces voltage unbalance even under balanced 3-phase loads.

System Consequences and Phase Transposition

Operational impacts of asymmetric conductor spacing:

- **Unbalanced Voltage Drops:** Positive-sequence currents (I_1) couple zero- and negative-sequence voltage drops ($V_0, V_2 \neq 0$);
- **Neutral and Earth Currents:** Continuous circulation of neutral currents through Carson ground return, elevating Joule losses;
- **Nuisance Protection Tripping:** Risk of unwanted ground overcurrent relay tripping (ANSI 51N) on long rural feeders.

Engineering Solutions

- **Phase Transposition Cycles:** Physical cable rotation at every 1/3 feeder length to cancel out average mutual couplings;
- **Compact Spacer-Cable Systems (NBR 15992):** Diamond-shaped spacers maintaining symmetrical 25 cm triangular geometry.

Engineering Conclusions and Unified Synthesis

Clayton Paul's multiconductor line theory unifies microwave and power systems:

1. **Triple Microstrip at 2.4 GHz:**
   - Inhomogeneous dielectric substrate causes modal dispersion ($\varepsilon_{\text{eff},1} = 2.94, \varepsilon_{\text{eff},2} = 3.20, \varepsilon_{\text{eff},3} = 3.71$);
   - Spatial directional beat enables 3 dB power splitters ($L_{\text{beat}}/4$) and directional couplers ($L_{\text{beat}}/2$).
2. **Overhead Distribution Lines at 60 Hz:**
   - Carson's ground return ($D_e = 850.6$ m) dominates mutual inductances;
   - Shifting the centre conductor increases L_{12} by 14.77% and doubles $|Z_{01}|$, requiring phase transposition or compact spacer cables (NBR 15992).

Official Bonus Material — Electromagnetics and RF Engineering Courses — UFES

National Institute of Science and Technology in Photonics (IN-Foton)

Telecommunications Laboratory (LabTel) — DEE / UFES

Open Educational Resources (OER) — Open Access

This educational material is licensed under a **Creative Commons Attribution-ShareAlike 4.0 International License (CC BY-SA 4.0)**.

Free to share, adapt, and reuse with mandatory attribution to: **Prof. Marcelo Segatto**, **LabTel / UFES**, and **IN-Foton**.

Canonical textbook figures and data cited under academic fair use with explicit sources.

Vitória, Espírito Santo, Brazil • 2026 • https://in-foton.org